Bonus Review:

Section 7:

(41) $\sum_{n=3}^{\infty} \frac{n}{n^2} = \sum_{n=3}^{\infty} \frac{1}{n} \rightarrow$ Diverges by p-series Test

(42) $\sum_{n=3}^{\infty} \frac{n}{n^2-4}$ vs. $\sum_{n=3}^{\infty} \frac{n}{n^2}$

(46) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ Since $\frac{1}{4} < 1$, the series converges by the Geometric Series Test.

(47) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ $\displaystyle\lim_{n\to\infty} \frac{(n+1)^2}{3^{n+1}} \cdot \frac{3^n}{n^2} = \lim_{n\to\infty} \frac{(n+1)^2}{3n^2} = \frac{1}{3}$

$\frac{1}{3} < 1$ so the series CONVERGES by the Ratio Test.

(48) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$ $\displaystyle\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n \rightarrow \lim_{n\to\infty} n \ln\left(1-\frac{1}{n}\right) =$

$\displaystyle\lim \frac{\ln(1-\frac{1}{n})}{\ldots} \overset{H}{=} \lim \frac{1}{1-n^{-1}} \cdot \cancel{n^2} \ldots =$

$\displaystyle\lim \frac{-1}{1} = -1$ $\displaystyle\lim \left(1-\frac{1}{n}\right)^n = \frac{1}{e}$

Section 8

39. if $\sum a_n^2$ converges then $\sum |a_n|$ converges

[illegible]

[illegible]

(55) Find the radius & interval of convergence:

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

Ratio Test →

$$\lim_{n\to\infty} \frac{\cancel{x}^{2n+2}}{(n+1)!} \cdot \frac{n!}{\cancel{x^{2n}}} =$$

$$\lim_{n\to\infty} \frac{x^2}{n+1} = 0 < 1$$

for all x's

So $\boxed{R=\infty}$ & Interval of Convergence is $\boxed{x\in(-\infty,\infty)}$

Section 10:

(31)

$$\int_0^1 \cos(x^2)\,dx$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$$

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}\,dx = \left[\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}\right]_0^1 =$$